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ROLE OF CALCULUS IN MATHEMATICS

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Abstract

The study of the different types of functions, the limitations associated with these functions, and how these functions change, together with the ability to graphically illustrate the basic concepts associated with these functions, are fundamental to computing. These important issues are presented along with the development of some additional elementary concepts that will help in our later studies to develop more advanced concepts. In this chapter and throughout this text, be aware that definitions and their consequences are the keys to success in understanding the calculation and its many applications and extensions. Note that Appendix B contains a summary of the fundamentals of algebra and trigonometry, which is a prerequisite for the calculation study. This first chapter is a preliminary to the calculation and begins with the introduction of the concepts of function, function graph and function limits. These concepts are introduced using some basic elements of set theory Because of the many relationships that link special functions to each other and to elementary functions, it is natural to consider whether more general functions can be developed in such a way that special functions and elementary functions are merely specialisations of these general functions. In fact, general functions of this nature have been developed and are collectively referred to as functions of a hypergeometric type. There are several varieties of these functions, but the hypergeometric functions are the most common.

Introduction

Technically, a student in the Calculus class is supposed to know both Algebra and Trigonometry. However, the reality is often very different. Most students enter the Calculus class, which is painfully unprepared for both the algebra and the trig in the Calculus class. This is very unfortunate because good algebra skills are absolutely vital to successfully completing any Calculus course and if your Calculus course includes trig (as this one does) good trig skills are also important in many sections. The aim of this chapter is to carry out a very brief review of some of the algebra and trig skills that are absolutely vital for a calculus course. This chapter is not inclusive of the algebra and trig skills needed to be successful in the Calculus course. It includes only those subjects in which most students are particularly deficient. For example, factoring is also vital to the completion of a standard calculus class, but is not included here.

Review: Functions – Here's a quick review of functions, a notation of functions, and a few fairly important ideas about functions.

Review: Inverse Functions – Quick review of inverse functions and notation of inverse functions.

Review: Trig Functions – Review of Trig Functions, Trig Function Evaluation and Unit Circle. This section is usually given a quick review in my class.

Review: Solving Trig Equations – a reminder of how to solve trig equations. This section is always covered by my class.

Review: Solving Trig Equations with Calculators, Part I – The previous section worked on the problem, the answers of which were always "standard" angles. In this section, we're working on some issues whose answers are not "standard" and so a calculator is needed. This section is always covered in my class, as most trig equations in the rest will require a calculator.

Review: Solving Trig Equations with Calculators, Part II – Even more trig equations that require a calculator to solve.

Review: Exponential Functions – Examination of exponential functions. This section is usually given a quick review in my class.

Review: Logarithm Functions – Review of logarithm and logarithm properties. This section is usually given a quick review in my class.

Review: Exponential and Logarithmic Equations – How to solve exponential and logarithmic equations. This section is always covered by my class.

Review: Common Graphs – There's not much in this section. It's mostly a collection of graphs of many of the common functions that could be seen in the Calculus class..

The Gaussian Hypergeometric Function and its Generalizations

John Wallis, in his work Arithmatica Infinitorum in 1655, first used the term 'hypergeometric' (from the Greek word above or beyond) to denote any series beyond the ordinary geometric series. $1 + x + x\ 2 + \cdots$ In particular, he studied the series $1 + a + a(a + 1) + a(a + 1)(a + 2) + \cdots$

Because of the many relationships that link special functions to each other and to elementary functions, it is natural to consider whether more general functions can be developed in such a way that special functions and elementary functions are merely the specialisations of these general functions. In fact, general functions of this nature have been developed and are collectively referred to as functions of a hypergeometric type. There are several varieties of these functions, but hypergeometric functions are the most common. Some important results concerning hypergeometric function were developed earlier by Euler and others, but it was the famous German mathematician C.F. Gauss who in 1812 studied the following infinite series, generalising the elementary geometric series and popularly known as the Gauss series or, more precisely, the Gauss hypergeometric series.

$$\sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n n!} x^n = 1 + \frac{a.b}{1.c} x + \frac{a(a+1).b(b+1)}{1.2.c(c+1)} x^2 + \cdots \quad \ldots(1.2.1)$$

Where

$$(a)_n = \prod_{k=1}^{n} (a + k - 1) = a(a+1)(a+2) \ldots (a+n-1)$$

is the Factorial function, or if $a > 0$ then $(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}$),Γ(where is Euler's Gamma function obviously $(a)0 = 1$ and $(a)n = n!$

Gauss represented this series by the symbol $F1(a, b; c; x)$ 2 and called it the hypergeometric function. Here x is a real or complex variable, a, b and c are parameters having $\neq($ 0, −1, −2, … .≠real or complex values and

$$ {}_2F_1(a,b;c;x) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n n!} x^n $$

…(1.2.2)

If c is zero or a negative integer, the function ${}_2F_1(a,b;c;x)$ is not defined unless one of the parameters a or b is also a negative integer such that $-c < -a$. If either of the parameters a or b is a negative integer, say – r, then in this case (1.2.1) reduces to the hypergeometric polynomial defined by

$$ {}_2F_1(-r,b;c;x) = \sum_{n=0}^{\infty} \frac{(-r)_n (b)_n}{(c)_n n!} x^n \ , \quad -\infty < x < \infty $$

…(1.2.3)

The series given by (1.2.1) is convergent when $|x| < 1$ and when $|x| = 1,$ provided that $Ree\ (c - a - b) > 0$ and also when $x = -1$, provided that $Re\ (c - a - b) > -1$.

In (1.2.1) if we replace x by $\frac{x}{b}$; let $b \to\infty$, then on taking into account the formula and

$$ \lim_{b\to\infty} \frac{(b)_n}{b^n} x^n = x^n $$

we arrive at the following well-known Kummer's series

$$ \sum_{n=0}^{\infty} \frac{(a)_n}{(c)_n n!} x^n = 1 + \frac{a}{c} x + \frac{a(a+1)}{c(c+1)} . \frac{x^2}{2!} + \cdots $$

…(1.2.5)

The above series is represented by the symbol $F1(a; c; x)$ 1 and known as Confluent Hypergeometric Function. The series given by (1.2.5) is absolutely convergent for all values of a, c and x, real or complex, excluding c = 0, −1, −2, … .

Gauss' hypergeometric function 2F1 and its confluent form 1F1 form the core of the special functions and include most of the commonly used functions as special cases. Thus 2F1 includes as its special cases the functions of Legendre, the incomplete beta function, the complete elliptical functions of the first and second types and most of the classical orthogonal polynomials. On the other hand, the confluent hypergeometric function 1F1 includes Bessel functions, parabolic cylinder functions, Coulomb wave functions, etc. as its special cases. Whittaker functions are also a slightly modified form of confluent hypergeometric function. Due to their usefulness, the functions 2F1 and 1F1 have already been explored to a considerable extent by a number of eminent mathematicians such as C. F. Gauss, E. E. Kummer, L. J. Slater, R. Mellin, E. W. Barnes, man.

Hypergeometric function 2F1 has been generalised by a variety of mathematicians, mainly in three ways:

(I) Increase the number of parameters;

(ii) Increase in the number of variables and

(iii) Increase the number of parameters as well as the variables.

(iv) The most common generalisation of the first type is the generalised hypergeometric function defined by the series

$$ {}_pF_q\left[\begin{matrix} a_1, \dots a_p; \\ b_1, \dots b_q; \end{matrix} \; x\right] = {}_pF_q[(a_p);(b_q);x] = \sum_{n=0}^{\infty} \frac{(a_1)_n \dots (a_p)_n}{(b_1)_n \dots (b_q)_n} \frac{x^n}{n!} \qquad \dots(1.2.6) $$

If p and q are positive integers or zero (interpreting empty product as 1) and we assume that variable x, numerator parameters a1, ... ap, and denominator parameters b1, ... bq are complex values, provided that

$$ b_j \neq 0, -1, -2, \dots; \; j = 1,2, \dots, q. $$

The application of the elementary ratio test to the power series on the right side of (1.2.6) shows that

- q; the series converges for all finite x,≤If p
- If $p = q + 1$, the series converges for $|x| < 1$ and diverges for $|x| > 1$.
- Furthermore, with $p = q + 1$, the series (1.2.6) is

(a) Absolutely convergent on the circle | x | = 1, if $Re(w) > 0$, where

$$w = \sum_{k=1}^{q} b_k - \sum_{k=1}^{p} a_k$$

...(1.2.7)

1, if – 1≠(b) Conditionally convergent for | x | = 1, < 0,≤$Re(w)$ and

(b) Divergent for | x | = 1 if Re (w) − 1≤

(i) If $p > q + 1$, the series never converges except when $x = 0$, and the function is only defined when the series terminates. A comprehensive account of $2F1$, $1F1$ and pFq functions can be found in the standard works by Slater Exton and Rainville . In attempt to give meaning to pFq in the case when $p > q + 1$, Mac Robert and Meijer introduced and studied in detail the two special functions that are well known in literature as the E-function and the G-function, respectively. A detailed account of the G-function is given in the works of Luke and Mathai and Saxen. . The E- and Gfunctions include wide variety of special functions as their particular cases. Though E- And Gfunctions are quite general in character, but still many functions like Wright's Generalized Hypergeometric Function, Wright's Generalized Bessel Function, Mittag-Leffler Function, and a number of other functions do not form their special cases..

Wright was given a generalisation of pFq in the following form:

$${}_p\psi_q\left[\begin{matrix}(a_1,\alpha_1),\ldots,(a_p,\alpha_p);\\ (b_1,\beta_1),\ldots,(b_q,\beta_q);\end{matrix}\; x\right] = {}_p\psi_q\left[\begin{matrix}(a_j,\alpha_j)_{1,p};\\ (b_j,\beta_j)_{1,q};\end{matrix}\; x\right] = \sum_{r=0}^{\infty} \frac{\prod_{j=1}^{p}\Gamma(a_j+\alpha_j r)}{\prod_{j=1}^{q}\Gamma((b_j+\beta_j r)}\frac{x^r}{r!}$$

...(1.2.8)

where $\alpha j$and βj (i = 1, … , p ; j = 1, … , q) are real and positive, and

$$1 + \sum_{j=1}^{q} \beta_j - \sum_{j=1}^{p} \alpha_j > 0$$

The Fox's H-Function and its Generalization

In 1961, Charles Fox introduced a more general function known in literature as Fox's H-function or simply the H-function. This function was defined and represented by the following type of integral contour of Mellin-Barnes.

$$H_{p,q}^{m,n}\left\{x\left|\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q}\end{matrix}\right.\right\} = H_{p,q}^{m,n}\left\{x\left|\begin{matrix}(a_1,\alpha_1),\dots,(a_p,\alpha_p)\\(b_1,\beta_1),\dots,(b_q,\beta_q)\end{matrix}\right.\right\} = \frac{1}{2\pi\omega}\int_L \theta(s)z^s ds \qquad \dots(1.3.1)$$

where $\omega = \sqrt{-1}, x \neq 0$ and

$$\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\prod_{j=m+1}^{q}\Gamma(1-b_j+\beta_j s)\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j s)} \qquad \dots(1.3.2)$$

Here m, n, p and q are non-negative integers satisfying 0 $j(j = 1, \dots , p)$ q. Also≤ $m \leq p$, 1 ≤ n and βj≤ (j = 1, … , q) are assumed to be positive quantities for standardization purpose. Also aj (j = 1, … , p) and bj (j = 1, … , q) are complex numbers such that

$$\alpha_i(b_h+\upsilon) \neq \beta_h(a_i-1-\eta) \qquad \dots(1.3.3)$$

for $\upsilon, \eta = 0,1,2,\dots\,;\ h = 1,\dots,m\,;\ i = 1,\dots,n.$

L is contour separating the points

$$s = \left(\frac{b_h+\upsilon}{\beta_h}\right); (h = 1,\dots,m\,; \upsilon = 0,1,2,\dots)$$

which are the poles of $\Gamma(b_h-\beta_h s); (h = 1,\dots,m),$ from the points

$$s = \left(\frac{a_i-1-\eta}{\alpha_i}\right); (i = 1,\dots,n; \eta = 0,1,2,\dots)$$

which are the poles of $\Gamma(1-a_i+\alpha_i s); i = 1,\dots,n.$ The contour L exists on account of (1.3.3). The series representation of the H-function is

$$H_{p,q}^{m,n}[x] = \sum_{h=1}^{m}\sum_{r=0}^{\infty} \frac{\prod_{\substack{j=1\\ j\neq h}}^{m}\Gamma(b_j - \beta_j\xi_{h,r})\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\xi_{h,r})}{\prod_{j=m+1}^{q}\Gamma(1-b_j+\beta_j\xi_{h,r})\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j\xi_{h,r})} \frac{(-1)^r (x)^{\xi_{h,r}}}{r!\,\beta_h}$$

...(1.3.4)

Where

$$\xi_{h,r} = \frac{b_h + r}{\beta_h}$$

...(1.3.5)

$$|\arg x| < \frac{1}{2}A\pi$$

...(1.3.6)

Where

$$A = \sum_{j=1}^{n}\alpha_j - \sum_{j=n+1}^{p}\alpha_j + \sum_{j=1}^{m}\beta_j - \sum_{j=m+1}^{q}\beta_j > 0$$

...(1.3.7)

A detailed discussion of the asymptotic expansion of the H-function, some of its properties and special cases may be referred to the books. In addition, some special functions provide direct solutions to several common and fractional order differential equations occurring in boundary value problems in various engineering fields. Such functions are Mittag-Leffler function, Agarwal function, Erdélyi function, Robotnov-Hartley F-function, Lorenzo-Hartley R-and G-functions, Miller-Ross Et, Ct and St-functions, Green reduced function due to Mainardi, Luchko and Pagnini. Gupta and Gupta and Soni have established a relationship between these highly useful named functions and Fox's H-function. Relationships are here

$$\frac{1}{\alpha t} H_{3,3}^{2,1}\left\{ x \left| \begin{matrix} \left(1,\frac{1}{\alpha}\right),\left(1,\frac{\beta}{\alpha}\right),(1,\rho) \\ \left(1,\frac{1}{\alpha}\right),(1,1),(1,\rho) \end{matrix} \right. \right\} = K_{\alpha,\beta}^{\theta}(t),$$

[Reduced Green function] ...(1.3.8)

$$\rho = (\alpha - \theta)/2\alpha,$$

$$\frac{t^{rq-v-1}}{\Gamma(r)} H_{1,2}^{1,1}\left\{-at^q \middle| \begin{matrix} (1-r,1) \\ (0,1),(1+v-rq,q) \end{matrix}\right\} = G_{q,v,r}[a,t]$$

[Lorenzo-Hartley G-function] ...(1.3.9)

$$t^v H_{1,2}^{1,1}\left\{-at \middle| \begin{matrix} (0,1) \\ (0,1),(-v,1) \end{matrix}\right\} = E_t[v,a]$$

[Miller-Ross E_t-function] ...(1.3.10)

$$t^v H_{1,2}^{1,1}\left\{a^2t^2 \middle| \begin{matrix} (0,1) \\ (0,1),(-v,2) \end{matrix}\right\} = C_t[v,a]$$

[Miller-Ross C_t-function] ...(1.3.11)

$$t^v H_{1,2}^{1,1}\left\{a^2t^2 \middle| \begin{matrix} (\frac{1}{2},1) \\ (\frac{1}{2},1),(-v,2) \end{matrix}\right\} = S_t[v,a]$$

[Miller-Ross S_t-function] ...(1.3.12)

$$t^{q-v-1} H_{1,2}^{1,1}\left\{-at^q \middle| \begin{matrix} (0,1) \\ (0,1),(1+v-q,q) \end{matrix}\right\} = R_{q,v}[a,t]$$

[Lorenzo-Hartley R-function] ...(1.3.13)

Since all special cases of R-function are Mittag-Leffler function, Agarwal function, Erdélyi function, Robotnov-Hartley F-function, these functions can be easily expressed in terms of H-function. The importance of the study of Fox's H-function lies in the fact that all the special functions referred to in the preceding paragraphs follow as their specific cases, so that each of the formulas developed for the H-function becomes a key formula from which a considerable number of relationships for other special functions can be derived by appropriately specialising the parameters of the H-function. A good collection of the work done on the H function can also be seen in the two books referred to above (see also). The H-function defined by (1.3.1) contains, as particular cases, most of the special functions mentioned above, but it does not contain some of the important functions, such as polylogarithm of a complex order, the exact partition function of the Gaussian model in statistical mechanics, etc..

Fractional Calculus

Fractional calculus originates from the question of the extension of meaning. For example, the extension of real numbers to complex numbers, the factorials of natural numbers to generalised factorials or gamma functions and many others. The original question that led to the name of the fractional calculus was as follows: Can the meaning of the derivative of the integer order? $\frac{d^n y}{dx^n}$ Is it extended to have a meaning when n is a fraction? Later the question became: can there be any number, fractional, irrational or complex? Since this question was answered in the affirmative, the name fractional calculus has become a misnomer and could be better referred to as integration and differentiation of an arbitrary order.

In 1974, the first international conference on fractional calculus was held at the University of New Haven, Connecticut, USA. Springer-Verlag published the proceedings of the conference. The second and third international conferences were held again in 1984 and 1989 at the University of Starthclyde, Glasgow, Scotland and Nihon University, Tokyo , Japan. There were many distinguished mathematicians attending these conferences. These luminaries have included R. Askey, M. Mikolas, M. Al-Bassam, P. Heywood, W. Lamb, R. Bagley, Y.A. Brychkov, R. Gorenflo, S.L. Kalla, E.R. Love, K. Nishimoto, S. Owa, A.P. Prudnikov, B. Ross, S. Samko, H.M. Srivastava, J.M.C. Joshi and many more. Papers on the fractional calculus and the generalised functions, inequalities resulting from the use of the fractional calculus and applications of the fractional calculus to the probability theory presented at the conference were quite electrical..

OBJECTIVES OF THE STUDY

1. To Study Role on Of Calculus In Mathematics
2. To Study on The Gaussian Hypergeometric Function and its Generalizations

CONCLUSION

The conclusions of the work presented in this thesis are set out in the following points-

- We have studied a pair of unified and extended fractional integral operators, including multivariate H-Function, I-Function and the general class of polynomials, which provide unification and generalisation of results to the various integral operators studied in recent years. In addition, some of the characteristics of these operators related to their Mellin transform have been investigated, generalising the scores of results to fractional integral operators scattered in the literature.
- The images of Saigo 's generalised fractional integral operators have been developed in terms of the product of I-function and the general class of polynomials. The results obtained here, in addition to being of a very general nature, were put in a compact form, avoiding the occurrence of infinite series and thus making them useful in applications.

- The integral formulas established provide for the extension and unification of a large number of results obtained earlier in the literature, thus enhancing the scope of their applications.
- Application of Sumudu and Laplace transforms to H-Sub-Function presents a comparative study of the results so obtained. The results obtained here are quite general in nature and are capable of producing a large number of results (new and known) in terms of simpler functions.

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